

Dilaton Quantum Gravity

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We propose a simple fixed point scenario in the renormalization flow of a scalar dilaton coupled to gravity. This would render gravity non-perturbatively renormalizable and thus constitute a viable theory of quantum gravity. On the fixed point dilatation symmetry is exact and the quantum effective action takes a very simple form. Realistic gravity with a nonzero Planck mass is obtained through a nonzero expectation value for the scalar field, constituting a spontaneous scale symmetry breaking. Furthermore, relevant couplings for the flow away from the fixed point can be associated with a “dilatation anomaly” that is responsible for dynamical dark energy. For the proposed fixed point and flow away from it the cosmological “constant” vanishes for asymptotic time.

During the recent years, much evidence has been collected for gravity to be asymptotically safe [1], allowing for a non-perturbative quantization [2]. Functional renormalization group methods [3, 4] have been used to show that the ultraviolet fixed point has remained rather robust for many extended truncations beyond Einstein-Hilbert gravity [5–9], thereby supporting and extending the results from various different investigations of the asymptotic safety scenario [10, 11]. Most recently, the attention was drawn to establishing a smooth trajectory from the ultraviolet regime towards the infrared limit [12–14]. The main ingredient of the fixed point is the ultraviolet scaling of the Planck mass proportional to the renormalization scale k .

For an exact fixed point dilatation, or scale, symmetry is exact. In general this entails an appropriate rescaling of the fields, usually including anomalous dimensions. The scale symmetry should be reflected in the quantum effective action in the limit where the infrared cutoff k is sent to zero. To date, the form of the quantum effective action associated with the fixed point in Einstein-Hilbert gravity is still disputed. Furthermore, in this theory the flow of a relevant parameter away from the fixed point is needed in order to end up with a non-vanishing Planck mass.

In this note we consider the system of a scalar field coupled to gravity. In the presence of a dilaton field χ it is straightforward to construct effective actions with dilatation symmetry [15–17]. We will argue that a corresponding rather simple fixed point is present in the functional flow equations for the dilaton coupled to gravity that we may associate with “dilaton quantum gravity”. This fixed point is associated to functions of $y = \chi^2/k^2$ that do not change during the flow. In this sense it involves infinitely many couplings of the dilaton-gravity system.

We investigate the functional flow within a simple truncation for the effective average action,

$$\Gamma_k = \int d^4x \sqrt{g} \left(V_k(\chi^2) - \frac{1}{2} F_k(\chi^2) R + \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi \right), \quad (1)$$

where R is the curvature scalar. The potential $V_k(\chi^2)$ and the coupling function $\frac{1}{2} F_k(\chi^2)$ are generic analytic functions of the square of the scalar field χ . The renormalization group flow of an action of the type (1) was first investigated in [9]. As a first approach, we use a simple kinetic term for χ , leaving the multiplication of the last term with a wave function renormalization $Z_k(\chi)$ for future work.

Our aim is a study of the flow of the functions V_k and

F_k as a function of the infrared cutoff $\sim k$. Effectively, this cutoff ensures that only quantum fluctuations with (covariant) momenta $|q| \gtrsim k$ are included in the computation of the effective average action Γ_k . In four dimensions, χ has dimension mass and we introduce dimensionless functions $v_k(y)$ and $f_k(y)$ through

$$V_k = k^4 y^2 v_k(y), \quad F_k = k^2 y f_k(y), \quad \text{where } y = \frac{\chi^2}{k^2}. \quad (2)$$

Our main suggestion is that the flow of the truncated effective action (1) has a fixed point for which the dimensionless functions v and f become independent of the renormalization scale k . They obey the asymptotic properties

$$\lim_{y \rightarrow \infty} f(y) = \xi \quad \text{and} \quad \lim_{y \rightarrow \infty} v(y) = 0. \quad (3)$$

Such a fixed point scenario has striking consequences. At the end, we are interested in the limit $k \rightarrow 0$, where all quantum fluctuations are included. For any nonzero value of χ , this corresponds to $y \rightarrow \infty$, such that the quantum effective action for the dilaton-gravity system takes the simple form

$$\Gamma = \int d^4x \sqrt{g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \frac{1}{2} \xi \chi^2 R \right). \quad (4)$$

This action is dilatation symmetric, as it should be for an exact fixed point. For any nonzero expectation value of χ dilatation symmetry is spontaneously broken, thereby generating a physical mass scale. The associated Goldstone boson is massless and will be referred to as the dilaton. The action (4) describes a viable theory of gravity.

Indeed, performing a canonical Weyl scaling of the metric $g_{\mu\nu}$ and using a rescaled scalar field ϕ , the effective action (4) becomes the Einstein-Hilbert action coupled to a massless scalar field, namely

$$\Gamma = \int d^4x \sqrt{g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} M^2 R \right). \quad (5)$$

The scale M characterizes the spontaneous breaking of dilatation symmetry, $\langle \chi \rangle = M/\sqrt{\xi}$. In this normalization M equals the reduced Planck mass, related to Newtons constant by $G_N^{-1} = 8\pi M^2$. Thus the physical content of the effective action (4) is Einstein gravity coupled to a massless dilaton. The dilaton plays no role in late cosmology [16], unless further “dilatation anomalies” are generated.

We are interested in the behavior of possible fixed point solutions for large y . In this region the properties of the fixed point are rather simple. The main ingredient will be the simple observation that for large y and nonzero

$$f_0 = \lim_{y \rightarrow \infty} f_k(y) \quad (6)$$

the strength of the gravitational interaction is given by $f_0^{-2} \chi^{-2}$. For the gravity induced flow of the dimensionless quantities only the dimensionless combination $f_0^{-2} y^{-1}$ can be of relevance. However, this quantity vanishes for $y \rightarrow \infty$ and the gravitational interactions are absent in this limit. For $y \rightarrow \infty$ and $v(y \rightarrow \infty) \rightarrow 0$ one is then left with a free scalar field. In turn, for a free scalar field the flow cannot induce a nontrivial χ -dependent effective potential, such that only a constant term can flow in V_k . For large y , the leading term in v_k is then proportional to y^{-2} and vanishes for $y \rightarrow \infty$, such that at the fixed point

$$\lim_{y \rightarrow \infty} v(y) = v_{-2} y^{-2}. \quad (7)$$

For a vanishing strength of the gravitational interaction the leading term in the gravitational sector of the effective action does not flow either. Thus f_0 does not depend on k , establishing the asymptotic behavior (3) with $\xi = \lim_{k \rightarrow 0} f_0$. Considering the fixed point (3) we will find that the dominant correction to F_k is a term $\sim k^2$, resulting in an asymptotic form for large y given by

$$\lim_{y \rightarrow \infty} f(y) = \xi + f_{-1} y^{-1}. \quad (8)$$

Flow equations. In order to quantify the results of the previous section, let us now turn to the explicit flow equations for the dimensionless functions $f_k(y)$ and $v_k(y)$. The flow at constant values of y takes the general form ($t = \ln(k/\mu)$)

$$\begin{aligned} \partial_t v_k(y) &= 2y v'_k(y) + \frac{1}{y^2} \zeta_V, \\ \partial_t f_k(y) &= 2y f'_k(y) + \frac{1}{y} \zeta_F. \end{aligned} \quad (9)$$

We employ the infrared cutoff introduced in [9], which is based on [18]. Then our computation of ζ_V and ζ_F yields in deDonder gauge and for the truncation (1)

$$\begin{aligned} \zeta_V &= \frac{1}{192\pi^2} \left\{ 6 + \frac{30\tilde{V}}{\Sigma_0} + \frac{3(2\Sigma_0 + 24y\tilde{F}'\Sigma'_0 + \tilde{F}\Sigma_1)}{\Delta} \right. \\ &\quad \left. + \delta_V \right\}, \\ \zeta_F &= \frac{1}{1152\pi^2} \left\{ 150 + \frac{30\tilde{F}(3\tilde{F} - 2\tilde{V})}{\Sigma_0^2} \right. \\ &\quad - \frac{12}{\Delta} \left(24y\tilde{F}'\Sigma'_0 + 2\Sigma_0 + \tilde{F}\Sigma_1 \right) - 6y(3\tilde{F}'^2 + 2\Sigma_0'^2) \\ &\quad \left. - \frac{36}{\Delta^2} \left[2y\Sigma_0\Sigma'_0(7\tilde{F}' - 2\tilde{V}')(\Sigma_1 - 1) + 2\Sigma_0^2\Sigma_2 \right] \right\} \end{aligned} \quad (10)$$

$$\begin{aligned} &+ 2y\Sigma_1(7\tilde{F}' - 2\tilde{V}') (2\Sigma_0\tilde{V}' - \tilde{V}\Sigma'_0) \\ &+ 24y\tilde{F}'\Sigma_0\Sigma'_0\Sigma_2 - 12y\tilde{F}\Sigma_0'^2\Sigma_2 \Big] + \delta_F \Big\}. \end{aligned}$$

Here we employ

$$\begin{aligned} \tilde{V} &= y^2 v_k(y), \quad \tilde{F} = y f_k(y), \\ \Sigma_0 &= \frac{1}{2} \tilde{F} - \tilde{V}, \quad \Delta = (12y\Sigma_0'^2 + \Sigma_0\Sigma_1), \\ \Sigma_1 &= 1 + 2\tilde{V}' + 4y\tilde{V}'', \quad \Sigma_2 = \tilde{F}' + 2y\tilde{F}'''. \end{aligned} \quad (11)$$

The contributions

$$\begin{aligned} \delta_V &= \left(\frac{4}{\tilde{F}} + \frac{5}{2\Sigma_0} + \frac{\Sigma_1}{2\Delta} \right) (\partial_t \tilde{F} + 2\tilde{F} - 2y\tilde{F}') \\ &\quad + \frac{12y\Sigma'_0}{\Delta} (\partial_t \tilde{F}' - 2y\tilde{F}'''), \\ \delta_F &= -\frac{\partial_t \tilde{F} + 2\tilde{F} - 2y\tilde{F}'}{\tilde{F}} \left[30 - \frac{5\tilde{F}(7\Sigma_0 + 4\tilde{V})}{\Sigma_0^2} \right. \\ &\quad + \frac{3}{\Delta^2} \left(\tilde{F}\Sigma_1\Delta + 8y\tilde{V}'\Sigma'_0\Delta - 24y\tilde{F}\Sigma_0'^2\Sigma_2 \right. \\ &\quad \left. \left. - 2y\tilde{F}\Sigma'_0\Sigma_1(7\tilde{F}' - 2\tilde{V}') \right) \right] \\ &\quad + \frac{6y}{\Delta^2} \left[(\tilde{F}' + 10\tilde{V}')\Delta - 24\Sigma_0\Sigma'_0\Sigma_2 \right. \\ &\quad \left. - 2(7\tilde{F}' - 2\tilde{V}')\Sigma_0\Sigma_1 \right] (\partial_t \tilde{F}' - 2y\tilde{F}'''), \end{aligned} \quad (12)$$

arise from the field dependence in the cutoff. They vanish for $y \rightarrow \infty$ and neglecting these contributions altogether does not change the structure of the results obtained. The flow equations (9) are in complete accordance to what was found in [9].

The flow generators ζ_V and ζ_F are expressed in terms of $v_k(y)$ and $f_k(y)$ as well as their first and second derivatives with respect to y , denoted by primes. They also involve y explicitly. The terms $2y v'_k(y)$ and $2y f'_k(y)$ arise in equations (9) when transforming from flow equations at fixed χ to the corresponding equations at fixed y , with $\partial_t y|_{\chi} = -2y$,

$$\frac{1}{\chi^2} \partial_t F_k|_{\chi} = \partial_t f_k|_{\chi} = \partial_t f_k|_y + f'_k \partial_t y|_{\chi}, \quad (13)$$

and similarly for $\partial_t v_k$. The functions ζ_V and ζ_F are defined as

$$\zeta_V = \frac{1}{k^4} \partial_t V_k|_{\chi}, \quad \zeta_F = \frac{1}{k^2} \partial_t F_k|_{\chi}. \quad (14)$$

We first investigate the generators ζ_V and ζ_F in the limit $y \rightarrow \infty$. In this limit we expand v and f in inverse powers of y

$$\begin{aligned} v(y) &= v_0 + v_{-1}y^{-1} + v_{-2}y^{-2} + \dots, \\ f(y) &= f_0 + f_{-1}y^{-1} + \dots \end{aligned} \quad (15)$$

Equations (10) yield

$$\lim_{y \rightarrow \infty} \zeta_V = \bar{\zeta}_V, \quad \lim_{y \rightarrow \infty} \zeta_F = \bar{\zeta}_F, \quad (16)$$

where the limits depend on f_0 and v_0 . We concentrate on

$$\begin{aligned} v_0 &= v_{-1} = 0, \quad \text{where} \\ \bar{\zeta}_V &= \frac{3}{32\pi^2} + \frac{5 + 33f_0}{96\pi^2(1 + 6f_0)} \frac{\partial_t f_0}{f_0}, \\ \bar{\zeta}_F &= \frac{77 + 534f_0}{192\pi^2(1 + 6f_0)} \\ &\quad + \frac{17 + 186f_0 + 720f_0^2}{576\pi^2(1 + 6f_0)^2} \frac{\partial_t f_0}{f_0}. \end{aligned} \quad (17)$$

We are interested in fixed point solutions $f^*(y), v^*(y)$ for which $\partial_t f_k(y) = \partial_t v_k(y) = 0$. The contributions $\sim \partial_t f_0$ vanish in this case. In the limit $y \rightarrow \infty$ equations (9) are then easily solved by

$$\lim_{y \rightarrow \infty} f^*(y) = \xi + \frac{\bar{\zeta}_F}{2y}, \quad \lim_{y \rightarrow \infty} v^*(y) = \frac{\bar{\zeta}_V}{4y^2}. \quad (18)$$

This coincides with the expectations (7), (8), with $v_{-1}^* = 0$ and $v_{-2}^* = \bar{\zeta}_V/4$, $f_{-1}^* = \bar{\zeta}_F/2$.

Scaling Solution. The most immediate global fixed point solution to the flow equations (9) is given by setting both \tilde{V} and \tilde{F} equal to a constant. This corresponds to Einstein-Hilbert gravity with an additional scalar field χ , which couples to gravity only through the metric $g_{\mu\nu}$. This solution obeys equation (9) exactly for all y and is numerically given by

$$\tilde{V} = y^2 v(y) = 0.008620 \quad \text{and} \quad \tilde{F} = y f(y) = 0.04751. \quad (19)$$

In order to gain insight into other classes of solutions, we have performed an expansion of $f^*(y)$ and $v^*(y)$ in powers of y^{-1} including the order y^{-8} . The result depends on the value ξ which is not fixed at this stage. The series shows excellent apparent convergence for $y \geq y_0 = 1/(10|\xi|)$. The convergence for smaller y can be improved by a Padé approximation, which we call $\tilde{V}_{\text{Padé}}$ and $\tilde{F}_{\text{Padé}}$, respectively. We show the result in figure 1, where numerator and denominator are expanded up to order y^{-4} . The Padé approximation satisfies the fixed point equations to good accuracy for $y > y_0 \approx 1/(100\xi)$.

We emphasize that the flow equations (9) are meaningful only if the relevant inverse propagators Σ_0 and Δ in the spin 2 and spin 0 sector remain positive for all y . These quantities correspond to the inverse graviton and scalar propagators in the presence of the cutoff k . For $y \rightarrow \infty$ one has $\Sigma_0 = \frac{1}{2}\xi y$, $\Sigma_1 = 1$, $\Delta = \frac{1}{2}\xi(1 + 6\xi)y$ such that Σ_0 and Δ are positive provided $\xi > 0$. We have checked that the positivity of Σ_0 and Δ also holds for the Padé approximation for the values of ξ shown in figure 1 and 2.

The positivity requirement for the propagators singles out asymptotic solutions for $y \rightarrow \infty$ for which $v_0 \leq 0$. In fact, the asymptotic fixed point solutions of equations (9)

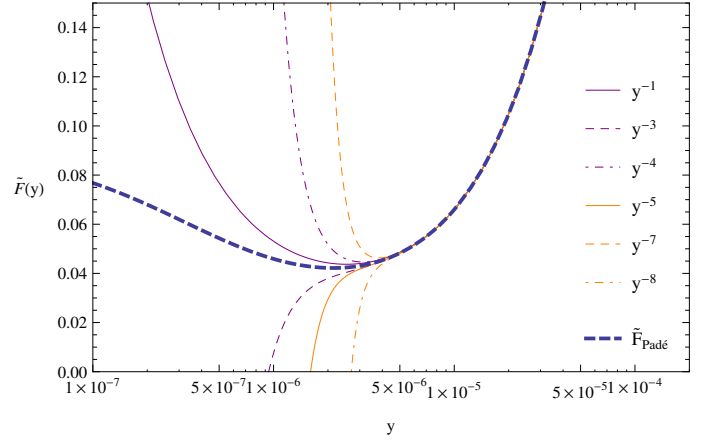


FIG. 1: Taylor expansions for $\tilde{F}(y)$ at $y = \infty$ with $\xi = 1000$, truncated at y^0 to y^{-8} and Padé improvement including powers of y^{-4} in both numerator and denominator. The splitting up of the Taylor series at the radius of convergence points to a breakdown of perturbation theory.

have also solutions with $v_0 \neq 0$, with

$$\begin{aligned} \bar{\zeta}_V &= -\frac{1}{48\pi^2} \left(6 - \frac{\partial_t f_0}{f_0} \right), \\ \bar{\zeta}_F &= \frac{1}{1728\pi^2} \left(249 - 41 \frac{\partial_t f_0}{f_0} \right), \end{aligned} \quad (20)$$

and corresponding values for the coefficients in the y^{-1} -expansion $v_{-2}^* = -0.00317$ and $f_{-1}^* = 0.0073$. (One also finds constant values for $\bar{\zeta}_V$ and $\bar{\zeta}_F$ in case of $f_0 = 0$. They differ from equations (20) and (17).) For $v_0 \neq 0$ the asymptotic form $\tilde{V} = v_0 y^2$, $\tilde{F} = \xi y$ implies a negative Σ_0 if $v_0 > 0$, rendering the propagation of the graviton unstable. With $\Sigma_0 = -v_0 y^2$, $\Sigma_1 = 12v_0 y$, one has $\Delta = 36v_0^2 y^3$ which remains positive for arbitrary $y \neq 0$.

Furthermore, we require that the potential V_k in our ansatz (1) is bounded from below in order to describe a stable theory. This holds for an asymptotic behavior with $v_0 \geq 0$. Combining the two requirements of a positive inverse propagator and a bounded potential only the asymptotic behavior $v_0 = 0$ is left. This absence of a term $\sim v_0 y^2$ is the crucial ingredient for the absence of a cosmological constant after Weyl scaling in (4).

The region of small $y \lesssim y_0$ is more difficult to access. One may investigate a Taylor expansion around $y = 0$ for the functions $\tilde{F} = y f^*(y)$ and $\tilde{V} = y^2 v^*(y)$,

$$\begin{aligned} \tilde{F} &= y f = F_0 + F_1 y + F_2 y^2 + \dots \\ \tilde{V} &= y^2 v = V_0 + V_1 y + V_2 y^2 + \dots \end{aligned} \quad (21)$$

The fixed point equations

$$\begin{aligned} \zeta_F - 2\tilde{F} + 2y\tilde{F}' &= 0, \\ \zeta_V - 4\tilde{V} + 2y\tilde{V}' &= 0, \end{aligned} \quad (22)$$

require that the constant terms in ζ_F and ζ_V equal $2F_0$ and $4V_0$, respectively. In turn, these constants $\zeta_F^{(0)}$ and $\zeta_V^{(0)}$ involve F_0, F_1 and V_0, V_1 . The system is not closed,

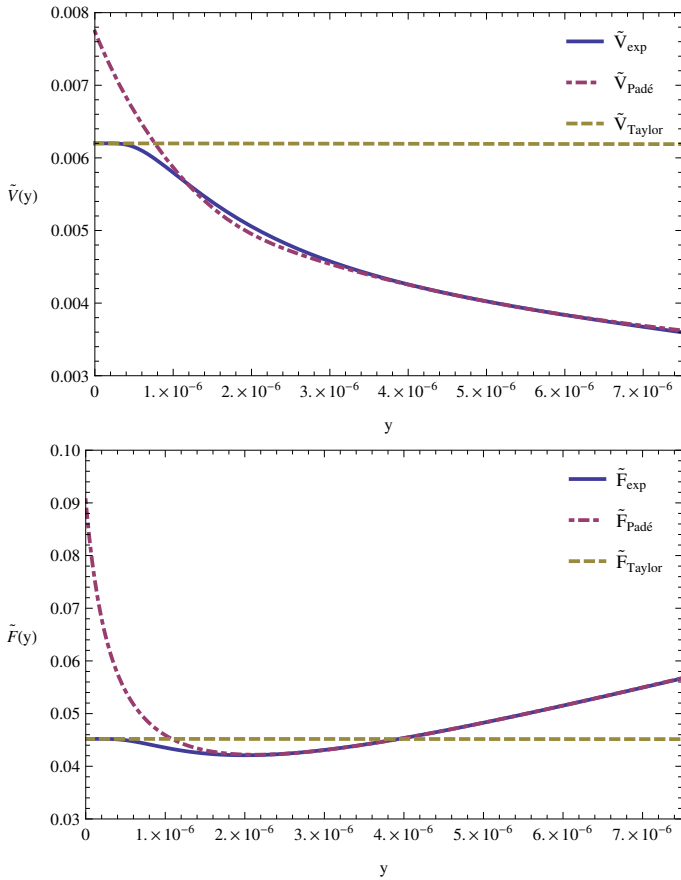


FIG. 2: Taylor expansion at $y = 0$ with $V_0 = 6.2 \times 10^{-3}$ and $F_0 = 2.3 \times 10^{-2}$, Padé approximation at $y = \infty$ with $\xi = 1000$ and the exponential ansatz (23) matched at $y = 5 \times 10^{-6}$ for the fixed functions $\tilde{V}(y)$ and $\tilde{F}(y)$.

and this property extends to higher orders in the Taylor expansion. For given F_0 and V_0 the Taylor expansion shows apparent convergence and we have expanded up to y^5 . We matched the Taylor expansion to the Padé expansion by the ansatz

$$\begin{aligned}\tilde{F}_{\text{exp}}(y) &= \tilde{F}_{\text{Taylor}}(y) + e^{-c/y} \tilde{F}_e(y), \\ \tilde{V}_{\text{exp}}(y) &= \tilde{V}_{\text{Taylor}}(y) + e^{-c/y} \tilde{V}_e(y),\end{aligned}\quad (23)$$

where $\tilde{F}_{\text{Taylor}}$, $\tilde{V}_{\text{Taylor}}$ are the Taylor expansions to order y^5 and \tilde{F}_e , \tilde{V}_e are polynomial functions of y to order y^2 . The coefficients of \tilde{F}_e and \tilde{V}_e are determined by matching the values and derivatives of F and V at some fixed $y = y_0$. We show the result in figure 2, for which we have determined F_0 and V_0 as well as the parameter c by varying their values around the limits of the corresponding Padé approximations for $y \rightarrow 0$ and optimizing the result as an approximate solution to equation (9).

The solution shown in figure 2 is only approximate. Further analysis will be needed to establish a global scaling solution. It is well conceivable that an exact fixed point exists only for one particular value of ξ . The true fixed point could also occur for a negative sign of the scalar kinetic term, or for a function $Z(y)$ multiplying this term which changes sign as a function of y . For the time being

we assume that an exact fixed point can finally be established and we explore further consequences of this scenario.

Flow Away from the Fixed Point. In contrast to the asymptotic freedom scenario in Einstein gravity there is no need to depart from the fixed point for the realization of a realistic model of gravity. In Einstein gravity the Planck mass is an intrinsic scale that violates dilatation symmetry. It is generated by the flow departing from the fixed point. In the language of critical phenomena the squared Planck mass corresponds to a relevant parameter. For a trajectory exactly on the fixed point the Planck mass would vanish. In contrast, in dilaton gravity a trajectory exactly on the fixed point can well describe gravity. Dilatation symmetry is then exact and this version of quantum gravity would correspond to a regularization that preserves exact dilatation symmetry [16, 19]. The Planck mass is generated by spontaneous dilatation symmetry breaking through the expectation value $\langle \chi \rangle$.

Nevertheless, the exact realization of the fixed point trajectory is not necessary. We may also discuss models of gravity where the renormalization flow starts very close to the fixed point in the ultraviolet, but the trajectory ultimately departs from the exact fixed point. This is analogous to asymptotic freedom in non-abelian gauge theories where the gauge coupling goes to zero for an infinite momentum scale (Gaussian fixed point), while for any finite arbitrarily large value of the scale it has a non-zero value. We will next show that this type of trajectory also leads to realistic gravity. It even entails an interesting cosmology with dark energy of the quintessence type.

We are interested in the range of large y or small $k^2 \ll \chi^2$. Deviations from the fixed point flow are most easily investigated by flow equations at fixed χ . A good approximation to the flow equations for V and F is then given by equation (14) with constant $\bar{\zeta}_V$ and $\bar{\zeta}_F$, namely

$$\partial_t V = \bar{\zeta}_V k^4, \quad \partial_t F = \bar{\zeta}_F k^2. \quad (24)$$

We look for solutions of equations (24) which respect the asymptotically constant values of ζ_V and ζ_F . Solutions of this type are simply

$$\begin{aligned}V &= \frac{\bar{\zeta}_V}{4} k^4 + \bar{V}, \\ F &= \xi \chi^2 + \frac{\bar{\zeta}_F}{2} k^2 + \bar{F}.\end{aligned}\quad (25)$$

Indeed, the χ^2 -independent integration constants \bar{V} and \bar{F} do not influence the value of $\bar{\zeta}_V$ and $\bar{\zeta}_F$. These constants contribute to ζ_V and ζ_F only in the order y^{-1} . This contrasts with the addition of a term $\bar{m}^2 \chi^2$ to the potential V . Even though this would formally still constitute a solution to equation (24), the values of $\bar{\zeta}_V$ and $\bar{\zeta}_F$ would be altered for the range $\bar{m}^2/k^2 > \xi$, since the asymptotic behavior of Σ_0 and Δ for $y \rightarrow \infty$ is modified. We observe that \bar{V} is a relevant parameter. The value of the dimensionless ratio V/k^4 is dominated by \bar{V} for $k \rightarrow 0$. In the limit $k \rightarrow 0$ the integration constant \bar{V} appears as a type of cosmological constant in the Jordan frame. Also \bar{F} remains important for $k \rightarrow 0$, but this is limited to $\xi \chi^2 \lesssim \bar{F}$. Both \bar{V} and \bar{F}

introduce explicit mass scales and break dilatation symmetry. For \bar{F} and \bar{V} different from zero the range of small χ with $\xi\chi^2 \lesssim \bar{F}$ resembles strongly the setting of Einstein-Hilbert gravity. The interesting new aspects of the present work concern the range $\xi\chi^2 \gg \bar{F}$.

In the presence of nonzero \bar{V} and \bar{F} the effective action of dilaton quantum gravity becomes for $k \rightarrow 0$

$$\Gamma = \int d^4x \sqrt{g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \frac{1}{2} (\xi\chi^2 + \bar{F}) R + \bar{V} \right), \quad (26)$$

generalizing equation (4). A cosmological constant in the Jordan frame has cosmological consequences that differ strongly from a cosmological constant in the Einstein frame [16]. This can be seen by a Weyl scaling

$$g'_{\mu\nu} = \frac{\xi\chi^2 + \bar{F}}{M^2} g_{\mu\nu},$$

combined with a rescaling of the scalar field

$$\varphi = M \ln \left(\frac{\xi\chi^2 + \bar{F}}{M^2} \right). \quad (27)$$

In terms of the new variables the effective action (26) reads

$$\begin{aligned} \Gamma &= \int d^4x \sqrt{g} \left\{ -\frac{M^2}{2} R + \frac{1}{2} k^2(\varphi) \partial^\mu \varphi \partial_\mu \varphi + V(\varphi) \right\}, \\ k^2(\varphi) &= \frac{1}{4\xi} \left[1 + 6\xi + \left(\frac{M^2}{\bar{F}} \exp \left(\frac{\varphi}{M} \right) - 1 \right)^{-1} \right]. \end{aligned} \quad (28)$$

The cosmon potential

$$V(\varphi) = \bar{V} \exp \left(-\frac{2\varphi}{M} \right) \quad (29)$$

decreases exponentially for large φ/M . Adding radiation and matter the cosmology described by the associated field equations admits for large φ/M a typical scaling solution with a fixed fraction of dynamical dark energy or quintessence. This holds for a negative sign of the scalar kinetic term in equation (1), with a suitable range of F for which the model remains stable. Our discussion can be extended to this case. For solutions where φ goes to infinity for asymptotic time the “cosmological constant” vanishes asymptotically.

We conclude that a fixed point with the asymptotic behavior (3) shows highly interesting properties. It would result in non-perturbatively renormalizable quantum gravity with a scale invariant quantum effective action (4). This action contains no term $\sim \chi^4$ due to $v_0 = 0$. The absence of such a term ensures the vanishing of the cosmological constant in the Einstein frame. Furthermore, deviations from exact scale invariance can lead to dark energy cosmology with an asymptotically vanishing “cosmological constant”. These interesting aspects may motivate the substantial work that is required in order to firmly establish such a fixed point.

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